Covariance Estimation

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Abstract

There exists a rich modern set of covariance matrix estimator methods for use in financial data. The purpose of `covmat` package is to implement some of these techniques such that they are readily available to be used with appropriate financial data. The purpose of this vignette is to demonstrate the usage of functions implemented in the `covmat` package.

Contents

1 Load Package

The latest version of the `covmat` package can be downloaded and installed through the following command:

```
library(devtools)
install_github("arorar/covmat")
```

The github version of covmat also implements Stambaugh and FMMC estimators which are not available in the CRAN release. The implementation of these estimators depened on the factorAnalytics package which is not yet available on CRAN.

2 Denoising using Random Matrix Theory

Random matrix theory provides a way to de-noise the sample covariance matrix . Let X be a matrix with T rows and N columns random matrix. C is the sample correlation matrix. Under the random matrix assumption, the eigenvalues of C must follow a Marchenko-Pastur density such that $N, T \to \infty, Q = N/T$. The density of eigenvalues is given by

$$f(\lambda) = \frac{Q}{2\pi\lambda\sigma^2}\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{\min})}$$

For a random matrix all eigenvaules will be within the range. The variance of these eigenvalues is 1. If any eigenvalue lies outside λ_{max} it is considered as a signal. We can choose these eigenvalues and replace the eigenvalues within the cutoff with either an average value or completely ignore them.

2.1 Data

To demonstrate the use of Random Matrix theory we will choose the `dow30data` object which contains daily returns for ow Jones 30 index for a year.

```
data("dow30data")
```

2.2 Covariance estimation

To fit a covariance matrix we can use the `estRMT` fucntion.

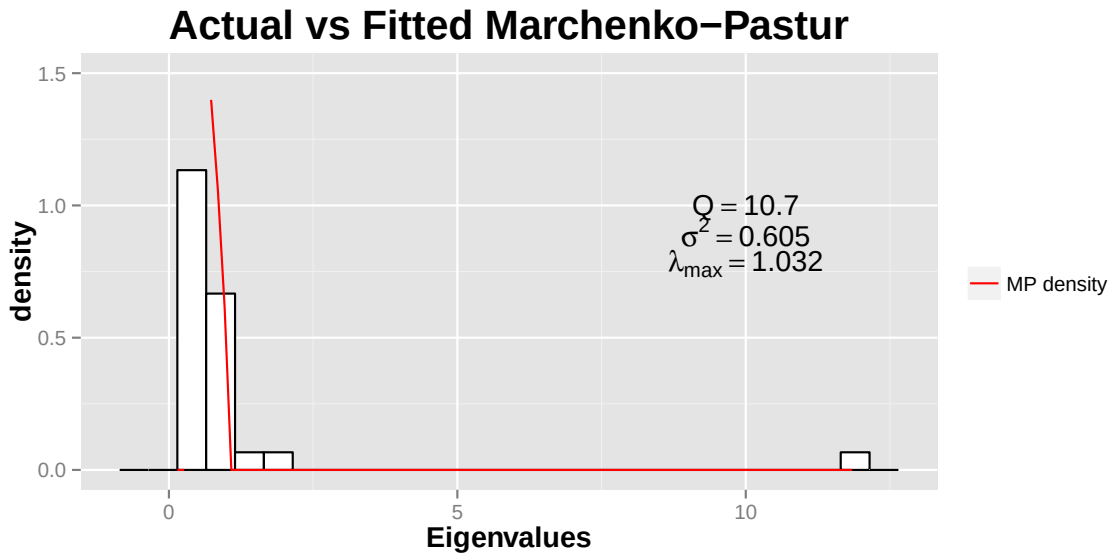
```
estRMT(R, Q =NA, cutoff = c("max", "each"),
       eigenTreat = c("average", "delete") ,
       numEig=1, parallel = TRUE)
```

This function takes serveral options, details of which can be found on the man page. However, in the simplest case we can pass a timeseries object of assets. In such a case we will assume that we know the largest eigenvalue and fit the distribution to the remaining eigenvalues. Values less than the cutoff are replaced with an average value.

2.3 Plots

Once we have fitted a model we can also investigate the fit visually using the `plot` function. The plot function takes in a fitted model and plots the fitted density overlayed on a histogram. It also displays some important fit parameters.

```
plot(model)
```



2.4 Evaluation

We will now demonstrate the use of RMT with a more elaborate example. Let us build a custom portfolio stratey using all 30 stocks from the Daily Dow Jones 30 index. We will use `dow30data` object that contains daily data from 04/02/2014 to 07/10/2015. We will use the `PortfolioAnalytics` package for building the portfolio and backtesting the strategy.

Let us first construct a custom moment function where covariance is built by denoising using Random Matrix Theory. We assume no third/fourth order effects.

```
custom.portfolio.moments <- function(R, portfolio) {
  momentargs <- list()
  momentargs$mu  <-  matrix(as.vector(apply(R,2, "mean")), ncol = 1)
  momentargs$sigma  <-  estRMT(R, parallel=FALSE)$cov
  momentargs$m3 <- matrix(0, nrow=ncol(R), ncol=ncol(R)^2)
  momentargs$m4 <- matrix(0, nrow=ncol(R), ncol=ncol(R)^3)

  return(momentargs)
}
```

We will construct a portfolio with the following specfication. No short sales are allowed. All cash needs to be invested at all times. As our objective, we will seek to maximize the quadratic utility which maximizes returns while controlling for risk.

```
pspec.lo <- portfolio.spec(assets = colnames(dow30data))

#long-only
pspec.lo <- add.constraint(pspec.lo, type="full_investment")
pspec.lo <- add.constraint(pspec.lo, type="long_only")

pspec.lo <- add.objective(portfolio=pspec.lo, type="return", name="mean")
pspec.lo <- add.objective(portfolio=pspec.lo, type="risk", name="var")
```

Now lets backtest our strategy using an ordinary covariance matrix and a covariance matrix build by denoising using Random Matrix theory.

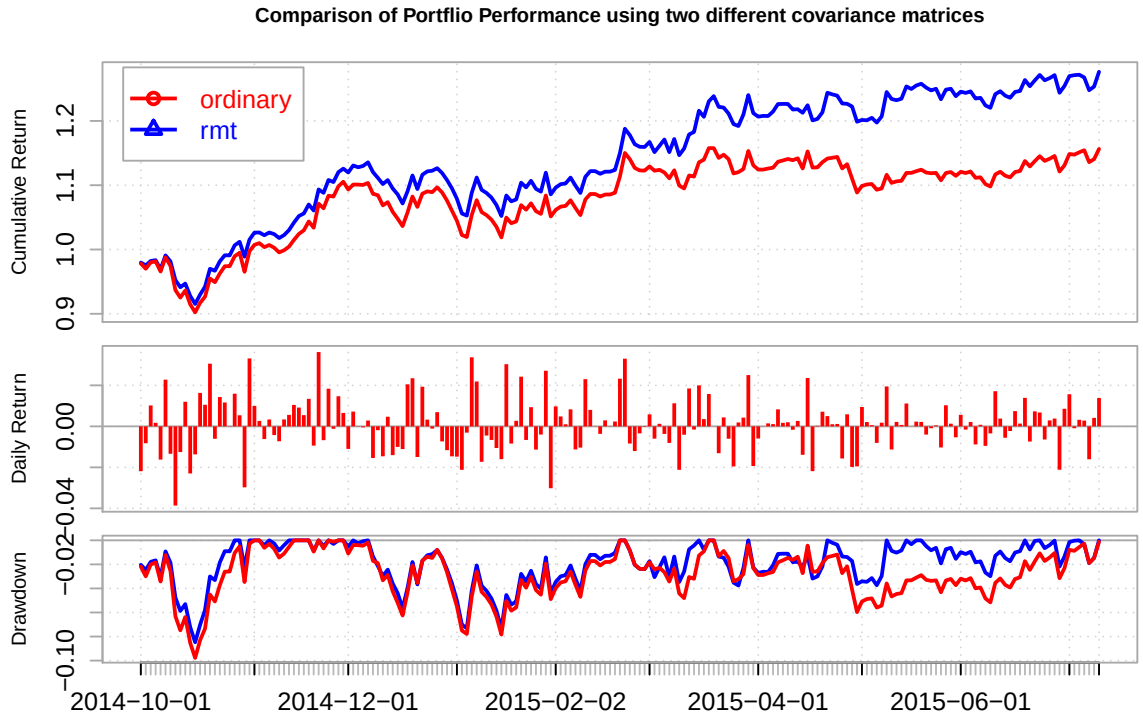
```
opt.ordinary <-
  optimize.portfolio.rebalancing(dow30data, pspec.lo,
                                 optimize_method="quadprog",
                                 rebalance_on="months",
                                 training_period=120,
                                 trailing_periods=120)
opt.rmt <-
  optimize.portfolio.rebalancing(dow30data, pspec.lo,
                                 optimize_method="quadprog",
                                 momentFUN = "custom.portfolio.moments",
                                 rebalance_on="months",
                                 training_period=120,
                                 trailing_periods=120)
```

We can now extract weights and build cummulative returns using the `PerformanceAnalytics` package.

```
ordinary.wts <- na.omit(extractWeights(opt.ordinary))
ordinary <- Return.rebalancing(R=dow30data, weights=ordinary.wts)

rmt.wts <- na.omit(extractWeights(opt.rmt))
rmt <- Return.rebalancing(R=dow30data, weights=rmt.wts)

rmt.strat.rets <- merge.zoo(ordinary,rmt)
colnames(rmt.strat.rets) <- c("ordinary", "rmt")
```

In the chart below we can see that the cumulative returns generated using our strategy with filtering using Random Matrix Theory are superior to ordinary returns. They are also better with smaller drawdowns. This suggests that there is value in filtering a large sample covariance matrix before using it for optimizing portfolios.

```
charts.PerformanceSummary(rmt.strat.rets,wealth.index = T,
                          colorset = c("red","blue"),
                          main=paste(c("Comparison of Portflio ",
                                       "Performance using two ",
                                       "different covariance matrices"),
                                     collapse=""), cex.legend = 1.3,
                          cex.axis = 1.3, legend.loc = "topleft")
```



3 Independent Switching Dynamic Conditional Correlation Model

The IS-DCC model from (Lee 2010) has the same structure as the DCC model but it lets the constants be state dependent and hence makes it possible to model time-varying correlation with different dynamics for each regime. The model runs a separate DCC process for each state in parallel and avoids the path dependency problem and makes the model tractable.

Fitting the IS-DCC model to data corresponds to a two step process, where the first step is to estimate the volatility of each univariate time series separately using GARCH(1,1), as in the case for DCC. The second step corresponds to estimating the DCC(1,1) parameters for each state and estimating the transition probabilities corresponding to the Hidden Markov Model. This is done by maximising the log likelihood using the Expectation Maximization (EM) algorithm.

3.1 Data

To demonstrate the use of IS-DCC we will choose the `etfdata` object which contains daily returns for 9 exchange traded funds (SPDRs) that represent the U.S. stock market from S&P, i.e. XLE, XLY, XLP, XLF, XLV,XLI, XLB, XLK and XLU. The Sector SPDRs divide the S&P 500 into nine sector index funds. The returns on assets are considered to be dependent on regimes which are in turn defined by market conditions. Daily returns on the ETFs from January 1, 2008 to December 31, 2010 were retrieved from Yahoo Finance.

```
data("etfdata")
```

3.2 Covariance estimation

To fit a covariance matrix we can use the `isdccfit` fucntion.

```
isdccfit(R, numRegimes, transMatbounds = c(2,10),
         dccBounds = c(0,1), w = NA, ...)
```

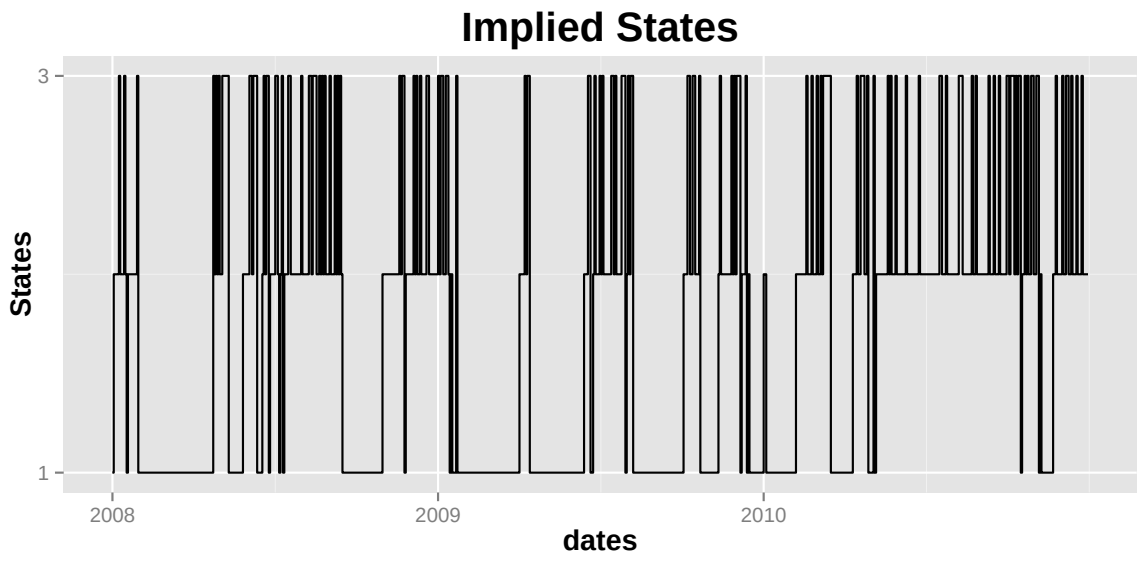
This function takes serveral options, details of which can be found on the man page. However, in the simplest case we can pass a timeseries object of asset returns and the number of regimes that we want to fit to the data. Since we use Expectation Maximization to fit data, convergence can be slow and additional parameters must be passed to the optimizer to speed by computation. We use `DEoptim` to fit paramters. If no control paramters are passed we use the `lhs` package to generate initial paramters uniformaly in the paramters space and pass it as initial population to `DEoptim`.

Let us fit a simple model for demonstration that fits three regimes to the data.

3.3 Plots

Once we have fitted a model we can also investigate the regimes by examining the implied states using the `plot` function. The plot function takes in a fitted model and and the type of plot. It then plots either the implied states or the implied probability of regimes.

```
plot(model.isdcc)
```



3.4 Evaluation

We will now build a custom portfolio stratey using `etfdata`. We will use a simple strategy such that at each period we will choose the covariance matrix from the regime which has the highest probability of occurance. We will benchmark the strategy against the ordinary DCC model that does not use regimes.

Let us first construct two custom moment functions where covariance is built using DCC and IS-DCC

```
custom.portfolio.moments.isdcc <- function(R, portfolio) {

  momentargs <- list()
  momentargs$mu  <-  matrix(as.vector(apply(R,2, "mean")), ncol = 1)

  result <- isdccfit(R, numRegimes = 2, itermax=100)
  ldate <- as.character(tail(index(R),1))
  maxProbIndex <- which.max(result$filtProb[ldate,])
```

```
  momentargs$sigma  <-  result$cov[[ldate]][[maxProbIndex]]

  momentargs$m3 <- matrix(0, nrow=ncol(R), ncol=ncol(R)^2)
  momentargs$m4 <- matrix(0, nrow=ncol(R), ncol=ncol(R)^3)

  return(momentargs)
}



custom.portfolio.moments.dcc <- function(R, portfolio) {

  momentargs <- list()
  momentargs$mu  <-  matrix(as.vector(apply(R,2, "mean")), ncol = 1)

  garch11.spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                             variance.model = list(garchOrder = c(1,1)
                                                   , model = "sGARCH"),
                             distribution.model = "norm")

  dcc.garch11.spec <- dccspec(uspec = multispec( replicate(ncol(R),
                                                 garch11.spec) ),
                              dccOrder = c(1,1), distribution = "mvnorm")

  dcc.fit <- dccfit(dcc.garch11.spec, data = R)
  momentargs$sigma  <-  rcov(dcc.fit)[,,as.character(tail(index(R),1))]

  momentargs$m3 <- matrix(0, nrow=ncol(R), ncol=ncol(R)^2)
  momentargs$m4 <- matrix(0, nrow=ncol(R), ncol=ncol(R)^3)

  return(momentargs)
}
```

We will construct a portfolio with the following specficatiion. No short sales are allowed. All cash needs to be invested at all times. As our objective, we will seek to maximize the quadratic utility which maximizes returns while controlling for risk.

```
datap <- etfdata["2009-07/"]

pspec.lo.isdcc <- portfolio.spec(assets = colnames(datap))

#long-only
pspec.lo.isdcc <- add.constraint(pspec.lo.isdcc, type="full_investment")
pspec.lo.isdcc <- add.constraint(pspec.lo.isdcc, type="long_only")
```

```
pspec.lo.isdcc <- add.objective(portfolio=pspec.lo.isdcc,
                                type="return", name="mean")
pspec.lo.isdcc <- add.objective(portfolio=pspec.lo.isdcc,
                                type="risk", name="var")
```

Now lets backtest our strategy using an ordinary covariance matrix and covariance matrices built using DCC and IS-DCC models.

```
ordinary <-
  optimize.portfolio.rebalancing(datap, pspec.lo.isdcc,
                                 optimize_method="quadprog",
                                 rebalance_on="months",
                                 training_period=120,
                                 trailing_periods=120)

opt.dcc <-
  optimize.portfolio.rebalancing(datap, pspec.lo.isdcc,
                                  optimize_method="quadprog",
                                  momentFUN =
                                   "custom.portfolio.moments.dcc",
                                  rebalance_on="months",
                                  training_period=120,
                                  trailing_periods=120)

opt.isdcc <-
  optimize.portfolio.rebalancing(datap, pspec.lo.isdcc,
                                         optimize_method="quadprog",
                                         momentFUN =
                                 "custom.portfolio.moments.isdcc",
                                         rebalance_on="months",
                                         training_period=120,
                                         trailing_periods=120)
```

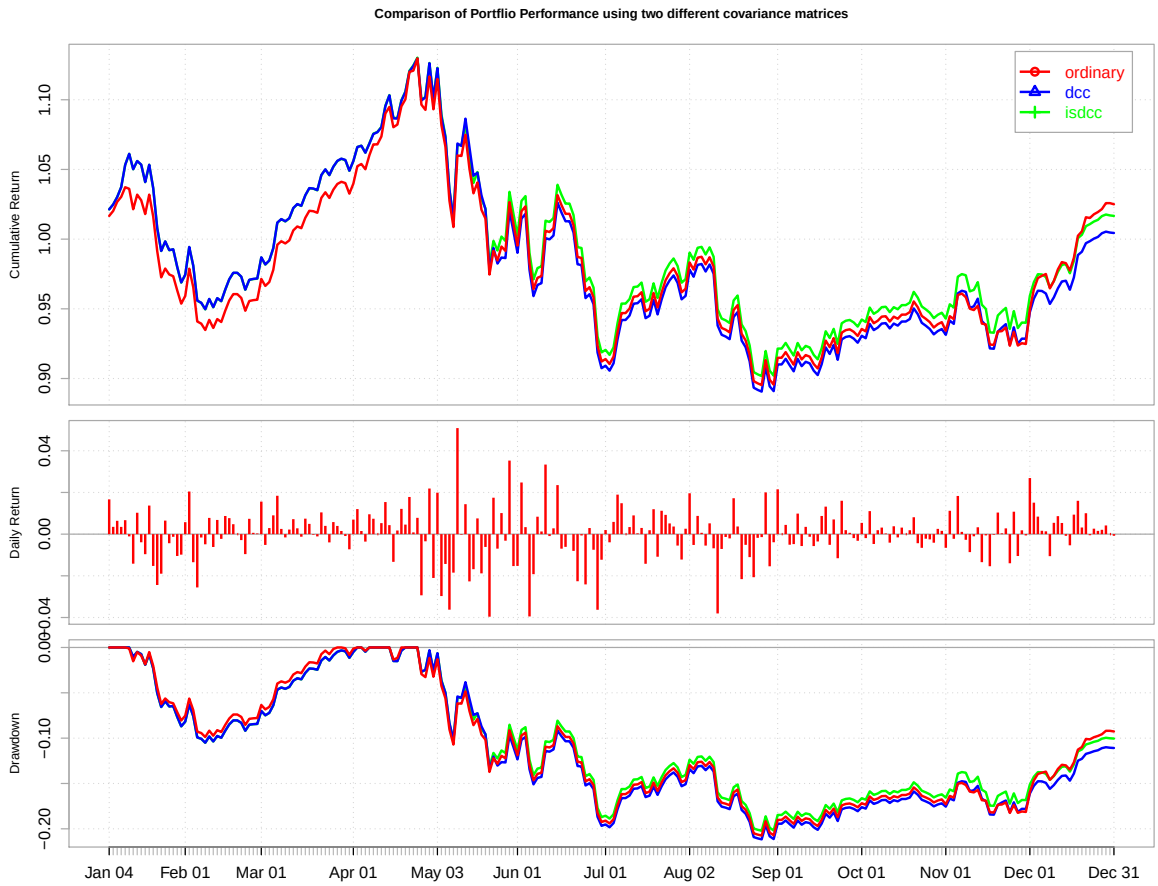
We can now extract weights and build cummulative returns using the `PerformanceAnalytics` package.

```
ord.wts <- na.omit(extractWeights(ordinary))
ord <- Return.rebalancing(R=datap, weights=ord.wts)

dcc.wts <- na.omit(extractWeights(opt.dcc))
dcc <- Return.rebalancing(R=datap, weights=dcc.wts)

isdcc.wts <- na.omit(extractWeights(opt.isdcc))
```

```
isdcc <- Return.rebalancing(R=datap, weights=isdcc.wts)

isdcc.strat.rets <- merge.zoo(merge.zoo(ord, dcc), isdcc)
colnames(isdcc.strat.rets) <- c("ordinary", "dcc", "isdcc")
```

In the chart below we can see that the cumulative returns generated using our strategy with IS-DCC model are superior to ordinary returns as well as returns by the DCC model. This suggests that there is value in assuming the presence of regimes in data and exploring the idea further while optimizing portfolios.

```
charts.PerformanceSummary(isdcc.strat.rets,wealth.index = T,
                          colorset = c("red","blue","green"),
                          main=paste(c("Comparison of Portflio ",
                                       "Performance using two ",
                                       "different covariance matrices"),
                                       collapse=""), cex.legend = 1.3,
                          cex.axis = 1.3, legend.loc = "topright")
```



4 Eigenvalue Shrinkage in Spiked Covariance Model

The MLE estimator of covariance matrix is not an accurate estimator when the ratio of number of variables to observations is large. It distorts the eigenstructure of the population covariance matrix such that the largest sample eigenvalue is biased upward and the smallest sample eigenvalue is biased downward. However, empirical eigenvalues can be improved by shrinkage. (Donoho, Gavish, and Johnstone 2013) assume that the population covariance matrix follows a spiked covariance model and construct scalar nonlinear shrinkers which shrink eigenvalues greater than the bulk edge of the Marchenko Pastur distribution and set values within bulk to 1. 26 loss functions under different losses and matrix norms are considered.

4.1 Data

To demonstrate the use of shrinkage in Spiked Covariance model we will choose the `rmtdata` object which contains simulated data with multivariate normal distribution. The data is generated as follows. We first generate a spiked covariance matrix with 100 dimensions and the following 15 eigenvalues as spikes, $\lambda \in \{48, 46, 44, 42, 40, 38, 36, 34, 32, 30, 28, 26, 24, 22, 20\}$.Our spiked covariance model is as follows

$$C = \sum_{\lambda_i \in \lambda} \lambda_i v_i v_i^\top + I_{100}$$

We will use the spiked covarianve matrix from above and generate 500 observations from a multivarite nornal distribution. This object is saved for ease of use and is called as `rmtdata`.

```
data("rmtdata")
```

4.2 Covariance estimation

To fit a covariance matrix we can use the `estSpikedCovariance` fucntion.

```
estSpikedCovariance(R, gamma = NA,
                       numOfSpikes = NA,
                       method = c("KNTest", "median-fitting"),
                       norm = c("Frobenius", "Operator", "Nuclear"),
```

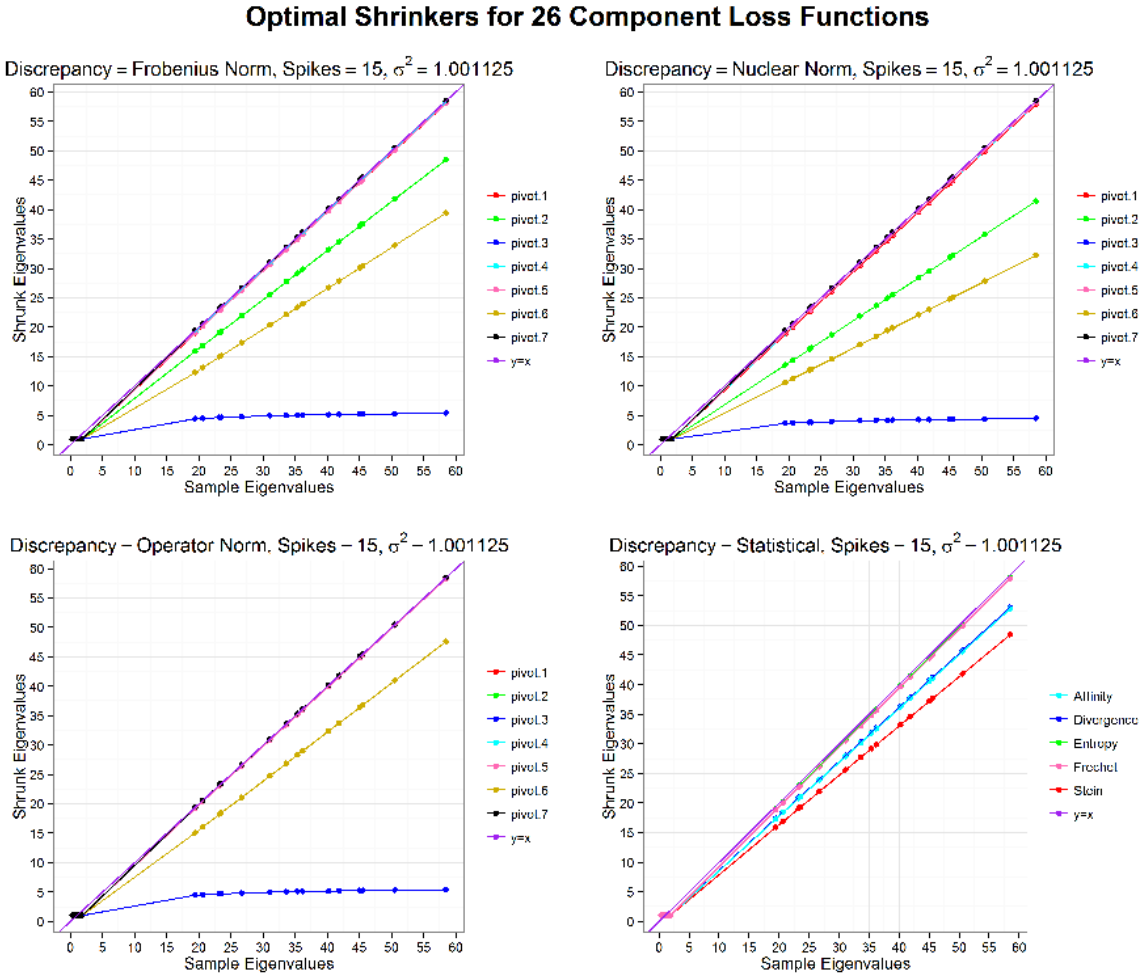
```
                        pivot = 1, statistical = NA,
                        fit = NA)
```

This function takes serveral options. In the simplest case we can pass a timeseries object of asset returns where all other parameters assume default value. The parameter gamma if missing is set to the ratio of variables to observations. For time series data, the choice of gamma can be important and one may want to control it such that the block of returns under consideration is stationary. If numOfSpikes is missing, then it is estimated using two methods, KNTest or median-fitting. In case of median-fitting we first count the number of breaks in the emperical histogram of eigenvalues using the Freedman-Diaconis algorithm. The initial number of spikes are calculated by counting the number of eigenvalues in the breaks after the first zero. The initial number of spikes are used to match the bulk edge and estimate σ^2. This serves as an lower bound for σ^2. σ^2 calculated by assuming no spikes is used as an upper bound. We then estimate variance by minimizing the absolute distance between the true median of the MP distribution and the median of the eigenvalues within the bulk. The details of KNTest desribed in (Kritchman and Nadler 2009).

4.3 Plots

We can investigate the peformance of 26 shrinkers by plotting them simultaneoulsy

```
plotSpikedCovariance(rmtdata)
```



5 Robust Exponential Smooting of Multivariate Time Series

Classical exponential smoothing is a popular technique used to forecast time series. However, presence of outliers can distort the forecasts leading to bad estimates. Robust methods result in much better forecasts than the classical approach in presence of outliers. (Croux, Gelper, and Mahieu 2010) suggest a methodology to combine these two techniques in a multivariate setting, where forecasting uses information from all components leading to more accurate forecasts.

5.1 Covariance estimation

To estimate a covariance matrix we can use the `estSpikedCovariance` fucntion.

```
robustMultExpSmoothing(R, smoothMat = NA, startup_period = 10,
                           training_period = 60 , seed = 9999, trials = 50,
                           method = "L-BFGS-B", lambda = 0.2)
```

The smoothing parameter is optional and will be estimated if missing. To estimate the smoothing matrix we use the constraints that the smoothing matrix is symmetric and its eigenvalues lie between 0 and 1. We also use the fact that the orthogonal matrix in spectral decomposition of smoothing matrix can be parameterized using givens angles. These angles must lie between $-\pi/2$ and $\pi/2$. To estimate the smoothing matrix we set up an optimization problem as described in the paper to minimize the determinant of the covariance of one step ahead forecast errors. The method argument allows one to change the optimization algorithm used. We use the `optimx` package to solve the multivariate optimization problem. The package allows us to choose from 16 different algoritms. Experimental results show that Nelder-Mead and L-BFGS-B perform well for such a noisy function. One also needs to be careful with the rerproducablility of the results. Esimation of smoothing matrix may lead to sligtly different results on each run. However, the cleaned series or covariance matrix show only marginal differnces. To estimate the matrix we start the optimization from random points and the parameter `trials` decides the number of runs and the parmater `seed` can be used for replicate the starting points. One needs to be careful with esitmation of smoothing matrix for high dimensional data. The estimation requires to search roughly, dimension squared parameters, which can be slow. Semidefinite-programming could lead to better solutions. However, at the time of writing the package, support for such a solver which is open source in R is challenging to find.

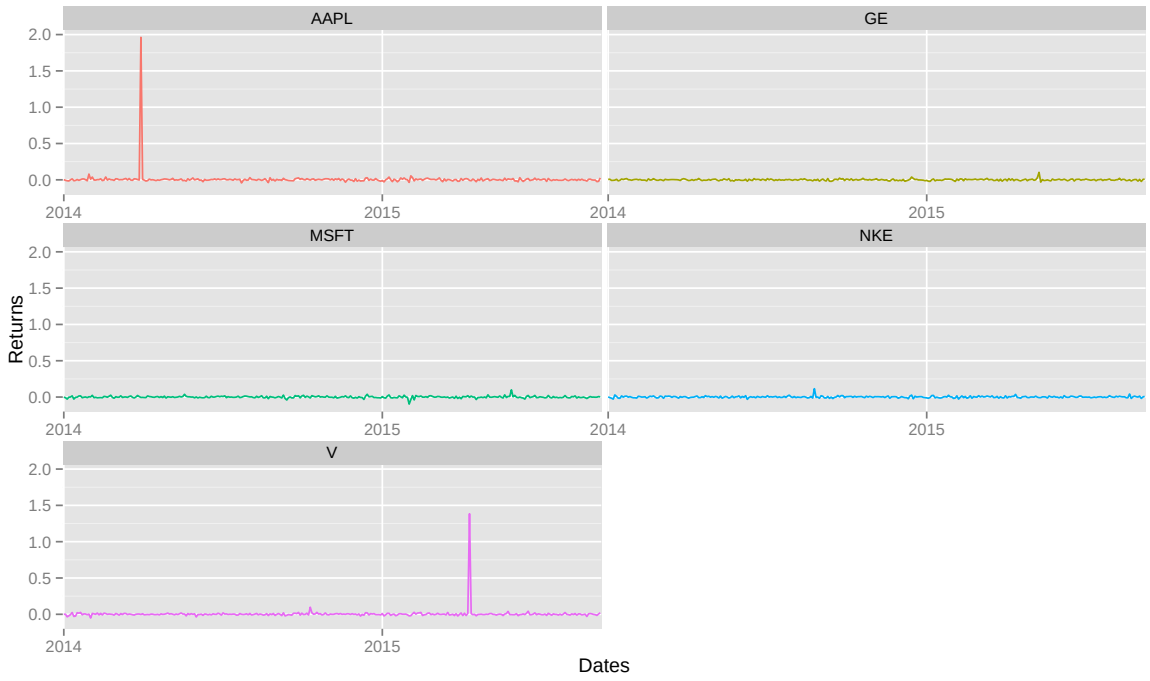
5.2 Evaluation

Let us use 5 stocks from Dow Jones 30 data, ie AAPL, GE, MSFT, NKE and V.

```
data("dow30data")
symbols <- c('AAPL', 'GE', 'MSFT' , 'NKE', 'V')

R <- dow30data[,which(colnames(dow30data) %in% symbols)]
smoothfit <- robustMultExpSmoothing(R)
```

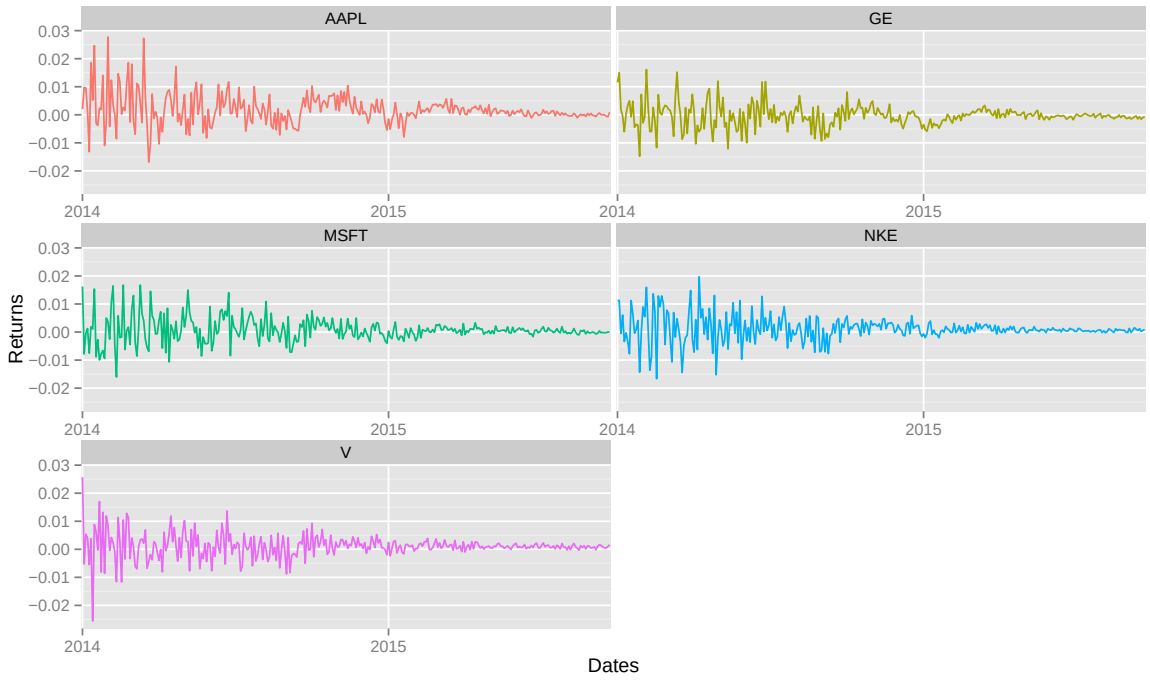
Now lets plot the returns. Notice the spikes in AAPL and V in the regular plot.

```
plotmissing(R)
```



Let us also examine the cleaned time series. Notice that the spikes are missing.

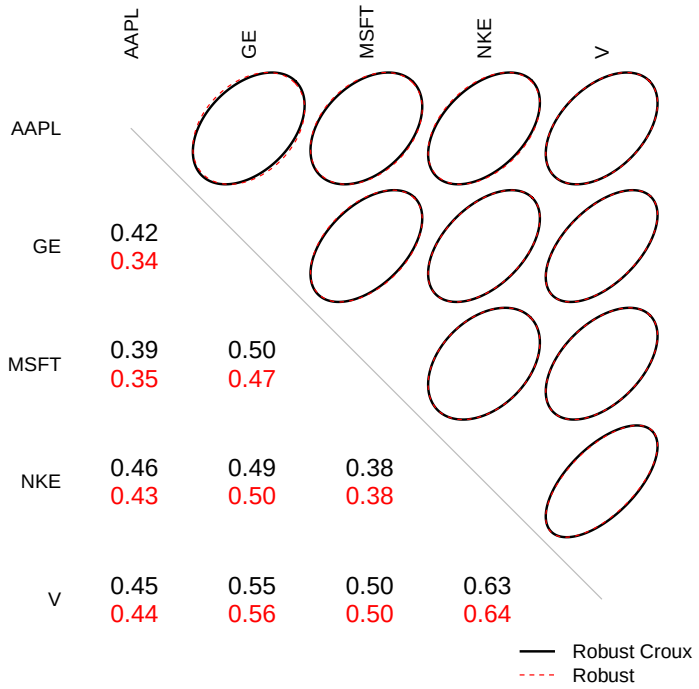
```
plotmissing(smoothfit$cleanValues)
```



Let us also compare the ordinary robust covariance matrix against robust covariance

matrix obtained using the procedure outlined in Croux. Notice the similarity between the two estimates.

```
rob <- covRob(R)$cov
compareCov(smoothfit$covMat, rob, labels = c("Robust Croux", "Robust"))
```



References

Croux, Christophe, Sarah Gelper, and Koen Mahieu. 2010. "Robust Exponential Smoothing of Multivariate Time Series." *Computational Statistics & Data Analysis* 54 (12). Elsevier BV: 2999–3006. doi:10.1016/j.csda.2009.05.003.

Donoho, David L., Matan Gavish, and Iain M. Johnstone. 2013. "Optimal Shrinkage of Eigenvalues in the Spiked Covariance Model." *Pre-Print*, November.

Kritchman, S., and B. Nadler. 2009. "Non-Parametric Detection of the Number of Signals: Hypothesis Testing and Random Matrix Theory." *IEEE Transactions on Signal Processing* 57 (10). Institute of Electrical & Electronics Engineers (IEEE): 3930–41. doi:10.1109/tsp.2009.2022897.

Lee, Hsiang-Tai. 2010. "Regime Switching Correlation Hedging." *Journal of Banking & Finance* 34 (11). Elsevier BV: 2728–41. doi:10.1016/j.jbankfin.2010.05.009.